THE SCALING THEORY
III. Interpretation of the Scaling Transformations of the First Type

C. P. Viazminsky

7. Interpretation of the Scaling Transformations

Two views of the same trip $(b(r,\theta,\emptyset)\ at\ B(R,\theta,\emptyset)) \to (O\ and\ o)$ are conceivable.

(i) To view the trip from a single frame, namely the frame in which the source is moving, which is *S*, and to look at the single light's trip $(b\, at\, B \to O)$ as two trips, of which only $(b \to O)$ is true while the other $(B \to O)$ is virtual. Here, *R* and *r* are the radial coordinates of *B* and *b* respectively in the frame *S*. This will be called the *active view*.

(ii) To view (*b at B*) as one single point in an absolute 3-physical space with one distance from (*O and* o). The symbols *R* and *r* in (7.4) are the coordinates of (*b at B*) in *S* and *s* respectively. This will be called the *passive view*.

The Active View

In case *b* is the source of light, the transformations (4.14) determine in the timed frame *S* the ratio between the characters of the true light trip $(b \to O)$ and the corresponding characters of the virtual light trip $(B \to O)$ that never took place. The radial coordinates *R* and *r* belong to *B* and *b* respectively. The values *r* and *t* are what *S* measure for the length and duration of the true light trip $(b \to O)$. If the theory is correct, these measured values must be related to the known geometric data *R* and *T* by the transformations

(6.1) $$r = \Gamma(u,\theta)R, \quad t = \Gamma(u,\theta)T = \Gamma(u,\theta)R/c.$$

If $T_0 = 0$ is read on the clock *B* at the instant of emission, then the time read on the clock *O* at the instant of light reception is

(6.2) $$t = \Gamma(u,\theta)\, R/c = r/c.$$

The quantity $T = R/c$ appearing in (6.2) represents the duration light takes from *B* to *O* were *B* a true source. Thus the *S* system of clocks alone is sufficient to specify the characters of the trip $(b \to O)$, since the reading of the clocks *B* and *O* of the events of emission and reception respectively determine these characters. Moreover and as evidenced by (6.2), the geometric distance *R* between *B* and *O* alone, is sufficient to determine the duration of the of the trip $(b \to O)$.

Note that, *R* is identified as the geometric distance between *O* and an *S* observer at the location $B(\vec{R})$ of the body *b* at the instant of light emission, while *r* is envisaged as the distance of the moving body *b* from the observer *O* when light is received by *O*. Equivalently, *r* is the geometric distance between *O* and an *S* observer *B*', that is contiguous to *b* when light is received. The latter distance, will be referred to as the *proper* (or mobile) distance. Thus the relations (4.14) give rise to transformations within the same frame *S*, between the geometric length *R* and the proper length *r* at the instant light is received by the observer *O*, whereas the directional coordinates are unaffected. In this interpretation, *R* can be assumed already given or known, while *r* which is calculable by the transformations (4.14) has to coincide with its measured value in *S* if the theory is really true. On account of (3.2) and (3.4) parallel statements hold for the geometric and proper durations *T* and *t* respectively.

Alternatively, the transformations (4.14) hold within s, with the rules of b and B as true and virtual sources are interchanged; R is the proper distance from o of a body B, which is moving with velocity $-u\vec{i}$ in s, and r is its geometric distance from o. Thus in the active view it is looked at the light's trip $(B\ at\ b \rightarrow o\ and\ O)$ as two trips of which one trip is true while the other is virtual. The proper quantities belong to the true trip whereas the geometric quantities characterize the virtual one. In the active view only one frame, namely the frame in which the source is moving is necessary for a full determination of the actual characters of the trip.

The Passive View

In the active view, no ambiguity arises regarding units, because the same units in one frame are used when considering the characters of the trips $(B \rightarrow O)$ and $(b \rightarrow O)$. If LS is the unit of length in S, then the geometric distance between two points may be written as $D = D_g.LS$. The numerical dimensionless quantity D_g is what is read by measuring instruments.

When two frames are involved in measuring the characters of the same trip, it is necessary beforehand to specify the units of length and time in each frame. We therefore interpret (4.14), as *defining units of length and time in S and s, whereas the characters of a true trip are absolute in a 3-physical space*. The absolute characters of the trip, which concern its length and duration, must be the same in both frames, as well as the already proven result regarding its directional angles. The latter statement is expressed as follows

- Length $\big((b\ at\ B) \rightarrow (O\ and\ o)\big) = Len$
- Duration $\big((b\ at\ B) \rightarrow (O\ and\ o)\big) = Tim$
- The direction of the trip is along the absolute segment connecting $(b\ at\ B)$ and $(O\ and\ o)$.

We denote the unit of length (time) in S and s by LS (TS) and ls (ts) respectively. Setting $R = LS, r = ls$ and $T = TS, t = ts$ in (4.14), we obtain

(7.1) $$\frac{ls}{LS} = \frac{ts}{TS} = \frac{\Gamma(u,\theta)}{1},$$

with $\vec{u} = u\vec{i}$ is the velocity of s relative to S. The latter relation dictates that the unit of length (time) in s and S must be in the proportion $\Gamma(u,\theta)\colon 1$. It is noted that if the unit of time in each frame is chosen equal to the duration taken by light to cross the unit length trip then $c = ls/ts = LS/TS = 1$ in both frames.

It is possible of course to adopt any chosen length (period) in a frame, say S, as the unit of length (time), but the choice of the unit of length (time) in other frame s must respect the proportion $1\colon 1/\Gamma(u,\theta)$. If the length of a given rod, or a specific wave length, is adopted as the unit of length in S, then there corresponds to this, the units $ls(u,\theta) = \Gamma(u,\theta)$ in s. Conversely, there correspond to a chosen length's unit ls in s the length's units $LS(u,\theta) = ls/\Gamma(u,\theta)$ in S. Units of length (time) in the frames s and S that obey the ratio $\Gamma(u,\theta)\colon 1$ will be referred to as a *system of relatively absolute units in S and s*, or just *absolute units*.

Having defined absolute units of length and time in S and s, the coordinates of the axes of S and s and the clocks in them should be calibrated accordingly. Since the length of a given light trip is absolute in the 3-physical space, we must have

(7.2a) $$Len = R_c.LS = r_c.ls$$

(7.2b) $$Tim = T_c.TS = t_c.ts$$

where R_c and T_c (r_c and t_c) are the time and duration of the trip as read in *S* (*s*) respectively. Employing (7.1) and (7.2a) we get

$$R_c.LS = r_c.(\Gamma(u,\theta)LS) = r_c\Gamma(u,\theta).LS$$

which yields

$$R_c = r_c\Gamma(u,\theta).$$

It follows that the inter-frames transformation between the *S* and *s* characters pertaining to a given light's trip have the expound form

(7.3) $$\frac{R_c}{r_c} = \frac{T_c}{t_c} = \frac{\Gamma(u,\theta)}{1}.$$

According to the last relations the observed characters in *S* are simply the *S* equivalents (means using *S* units) of the corresponding observed characters in *s*, and vice versa. We shall often drop the index *c* and write (7.3) in the form

(7.4) $$\frac{R}{r} = \frac{T}{t} = \frac{\Gamma(u,\theta)}{1},$$

provided we understand the symbols appearing in the last equations as referring to the characters' readings of the same light trip in *S* and *s*.

Note that the relations (7.4) determine only the ratio between the values of the same character (length or duration) in two frames. To determine both values, this character must be measured in either frame. However, the universal point (*b* at *B*) originates from a source *b* moving in *S* and its location *B* in *S*. The distance of *b* from (*O* and *o*) is its geometric distance. Hence

$$r = r_g ls = Len = r_c ls,$$

and (7.4) takes the form

$$\frac{R}{r_g} = \frac{T}{t_g} = \Gamma(u,\theta).$$

If *B* is the source then

$$\frac{R_g}{r} = \frac{T_g}{t} = \Gamma(u,\theta).$$

It is noted that both frames assign to a single light's trip its geometric characters in the frame at which the source is stationary; these however, have different readings due to the usage of different units.