

## On Contiguous Inertial Frames

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### Abstract

In earlier works (Viazminsky, 2002, 2003) the concept of contiguous inertial frames was introduced. It was shown that the equivalence of an inertial frame and the set of frames that are contiguous to it requires on one hand introducing in addition to the absolute intervals of time and distance counter moving quantities, i.e. moving time intervals and distance, and on the other hand adopting certain transformation between the absolute and moving quantities. The latter transformations yield as a result the velocity of light constant and independent of the relative motion between the source and the observer. The Doppler, drag, and aberration effects were explained within the frame work of this approach. It is shown here that the current approach yield the same dynamical laws of relativity including the well-known mass-energy equivalence relation.

### 1. Introduction

In an earlier work [1,2] the concept of contiguous inertial frames was introduced. The consequences of incorporating a constant velocity of light signals 'within' any inertial frame was utilized to derive certain relations between the coordinates of a moving body in the two contiguous frames. This gave rise to types of coordinates, the stationary and moving coordinates. In this work we shall elaborate on the previous concepts and results spelling out the implications of adopting the moving time as the proper time that should be associated with motion, on the forms of basic dynamical entities, such as mass, momentum, energy and force.

The current work starts with one single assumption: the velocity of light within each inertial frame equals to a constant  $c$ . By this we shall mean that if light is emitted at an instant  $t_0$  from the point  $\vec{r}_0$  and received at an instant  $t$  at the point  $\vec{r}$ , where the points  $\vec{r}_0$  and  $\vec{r}$  are stationary in the inertial frame  $S$ , then  $|\vec{r} - \vec{r}_0| / (t - t_0) = c$ . We have assumed implicitly that clocks in each inertial frame can be synchronized by the usual procedure of light signals (Rindler, 1977 ; Mould, 1994) so that time is absolute in each inertial frame. This latter assumption regarding the constancy of the light's velocity within each inertial frame may hardly be counted as a postulate. It may be instead, considered as a natural consequence that follows from the equivalence of inertial frames as regard their geometric and temporal structure. In comparison, the special theory of relativity assumes that the velocity of light emitted from a source is the same in all inertial frames, and thus, is independent of the state of relative motion between the source of light and the observer. In our assumption the source is stationary relative to the observer. For start we shall apply the Galilean law of velocity addition to light signals emitted from a moving source and show that this results in a contradiction, which requires for its elimination adopting a new transformation on one hand and introducing two types of time intervals on the other hand. It will be shown that for a phenomenon that requires measurements occupying a finite time interval the symmetric statue between two contiguous observers fads away leaving the permanent observer as privileged, with his own

time as absolute, while the second type of time is the proper time to be associated with light signal as well as with the motion of the body. It will turn out that all inertial observers have full agreement on the absolute time in the sense that every clock will register the same 'time intervals' disregarding their state of motion. The change in magnitude applies only to 'inter-bodies' or 'signals' time intervals, and hence there will be no twin or clock paradox.

## 2. The Concept of Inertial Contiguity

As inertial contiguity is a basic concept in the development of the forthcoming work, the current section will be devoted to introducing and explaining this concept through a simple example. Imagine a train passing by a station deck with a uniform velocity  $\vec{u}$ . Let  $S \equiv (S, o) \equiv oxy$  be a rectangular Cartesian frame of coordinates attached to the deck, with  $o$  is a fixed point of the deck,  $ox$  is along the train's motion and  $oy$  is in the deck's plain. Let  $O$  be a point of the train that coincides with  $o$  at  $t = 0$ , and  $M \equiv (M, O) \equiv OXY'$  be a rectangular Cartesian system of coordinates fixed to the train and in standard configuration with  $S$ . The Galilean transformations between  $S$  and  $M$  are

$$(2.1) \quad x = X' + ut, \quad y = Y', \quad t = T$$

where, in accord with Newtonian mechanics, time has been considered for start as an absolute entity. Let  $B$  be a point with coordinates  $(x, y)$  in the deck frame  $S$ . Instead of specifying the coordinates of  $B$  relative to the train frame  $(M, O)$  i.e. relative to the train frame with origin at  $O$  (which coincides with  $o$  at  $t = 0$ ), we specify it at each instant  $t$  relative to the frame  $M_t \equiv (M, O_t)$ , which is a frame moving with the train, but it has its origin at a point  $O_t$  that is contiguous to  $o$  at the instant  $t$ . According to this way of coordinatization, an infinite set of adjacent inertial frame all moving with the train is employed, and of which one at a time is used to specify the coordinates of the point  $B$ . If the coordinates of  $B$  are  $(x, y)$  in  $S$  and  $(X, Y)$  in  $M$  then at any instant  $t$

$$(2.2) \quad x = X, \quad y = Y, \quad t = T$$

The latter relations are valid whether  $B$  is at rest or motion relative to  $S$ . The Galilean transformations are recovered on noting that the coordinates of  $O_t$ , or  $o$  with respect to  $M \equiv M_t$  are  $(-ut, 0)$ , so that

$$(2.3) \quad x(t) = X(t) = X' + ut, \quad y = Y, \quad t = T.$$

A hasty look may conclude that the previous considerations are redundant, for, the set of frames  $\{M_t\}$  results from  $M$  through the translations  $X' \rightarrow X' + ut = x$ , and hence are equivalent to a single frame coinciding with  $M \equiv M_0$  at  $t = 0$  and moving relative to the train with velocity  $(-u)$ , which is precisely the frame  $S$ . This view gains additional ground on noting that the measurements  $\{(\vec{R}, t) = (X, Y, t)\}$  gathered by the set of observers  $\{M_t\}$  is identical to the set  $\{(\vec{r}, t) = (x, y, t)\}$  gathered by  $S$ , and consequently, the velocity and acceleration are the same. Since mass is invariant in Newtonian mechanics, the force exerted on a material point is also the same. This apparent coincidence between  $S$  and  $\{M_t\}$  is deceptive, for there is still a major difference which is manifested in the state of motion of every particle as observed from  $S$  and each  $M_t$  separately. Indeed, and if the state of motion of a particle  $B$  is characterized by  $(\vec{r}, \vec{v})$  in  $S$ , then its state at the same instant  $t$  is characterized by  $(\vec{R}, \vec{V})$  in  $M_t$ , where

$$(2.4) \quad \vec{R} = \vec{r}, \quad \vec{V} = \vec{v} - \vec{u},$$

The inverse transformation is  $\vec{r} = \vec{R}, \vec{v} = \vec{V} + \vec{u}$ . If  $B$ , for example, is a fixed point of the train, then its velocity relative to  $S$  is  $u$ . Although  $B$  is at rest relative to each frame  $M_t$ , its velocity with respect to the system of frames  $\{M_t\}$  is  $dx/dt = dX/dt$ . We may imagine  $B$  is a point of a train that is composed of extremely tiny compartments, as tiny as we please, and the position of  $B$  is determined at each instant with respect to the compartment that is

contiguous to the point  $o$  of the deck. The point  $B$  is at rest relative to each compartment contiguous to  $o$  but has velocity  $u$  when more than one compartment is involved. On the other hand, a particle which is at rest in the deck frame, has velocity  $(-u)$  relative to each compartment contiguous to  $o$ , but is stationary when many compartments are involved.

One may imagine that each  $M_t$  inherits his predecessors' measurements and that  $M_t$  is activated only within a small time interval corresponding to the particle passing in a small neighborhood of  $o$ . We may thus talk at each instant  $t$  of a single frame  $M_t$  instead of the set of frames  $\{M_s : s \leq t\}$ . Within this view there are two types of velocities in  $M_t$ :

(i) the orbital velocity of a particle which is the usual derivative of the position vector with respect to time:

$$\vec{v} = \frac{dR}{dt} = \frac{d\vec{r}}{dt}.$$

This quantity is the same in both frames.

(ii) the actual velocity

$$\vec{V} = \vec{v} - \vec{u} = \frac{d\vec{r}}{dt} - \vec{u}$$

The relations (2.4) shows that the actual velocity transformation between  $S$  and one  $M_t$  is thus the same as that in Galilean transformation in spite of the fact that the trajectory of the particle is also the same in both frames.

As an example we consider a particle  $B$  whose trajectory in  $S$  is

$$x = v_x(t - t_0), \quad y = v_y(t - t_0),$$

which is a segment of a straight line starting at  $t = 0$  from the point  $(x = -t_0 v_x, y = -t_0 v_y)$  and passing at  $t = t_0$  from  $o$ . Though the same trajectory is observed from  $M_{t_0}$ , the velocity of the particle at any point is  $\vec{v} = (v_x, v_y)$  in  $S$  and  $\vec{V} = (v_x - u, v_y)$  in  $M_{t_0}$ . It is important to note that the direction of the velocity of a particle as observed in  $M_t$  is different from that observed in  $S$ . This fact will be used in section 6 in [1] to explain the aberration phenomenon, which in essence is the existence of an angle between the velocities of any moving object ( a beam of light in aberration) in  $S$  and  $M_t$  [2].

## Definition of Contiguous Frames

Having introduced the concept of inertial contiguity we elaborate on this concept through the mathematical definition.

Definition: Let  $(S, o)$  be an inertial frame and  $\{(M, O_t) \equiv M_t\}$  be a family of inertial frames that have the properties:

- (i)  $O_t$  is coincident with  $o$  at the instant  $t$ , and  $oO = ut\vec{i}$ .
- (ii) At each instant  $t$ , the measurements of the position of the particle and the corresponding time are made with respect to  $M_t \equiv (M, O_t)$ .
- (iii) Each frame  $M_t$  inherits his predecessors  $\{M_s : s \leq t\}$  measurements

Then each frame  $M_t$  will be called a  $u$ -contiguous frame to  $S \equiv (S, o)$  at the instant  $t$ . We shall describe  $O_t$  as a contiguous observer to  $o$  at the instant  $t$ .

It is clear that if  $M$  is  $u$ -contiguous to  $(S, o)$  at  $t=0$ , then on considering  $(M, O)$  stationary, the set of frames  $S_t \equiv (S, o_t)$  is  $(-u)$ -contiguous to  $M \equiv (M, O)$  at  $t$ . We may thus say that  $O$  and  $o$  are contiguous observers at  $t = 0$ . When two observers are contiguous at some instant, say  $t = 0$ , each can count his fellow observer as a "transit" or temporal observer, i.e. to be replaced soon by another, and claim himself as a "prime" or permanent observer, i.e. with respect to his location the measurements of position vectors of a particle have taken place and will continue to be made. In other words, we are at liberty to consider either observer as prime and the other as transit. Indeed, the transformation (2.4) are completely symmetric with respect to the contiguous observers, i.e. it can't reveal which observer is prime and hence the position measurements should always be made with respect to his own location (origin), and which is transit, so that his origin has to be replaced by another.

We shall see later that introducing a finite speed for information or signals to measure positions, distorts the absolute nature of time and distance, giving rise to two types of times

- (i) 'Internal' or absolute time, which is related to the internal processes within a body, or to signals between points that are stationary with respect to each other (static space).
- (ii) 'Inter-bodies' or 'signals' time, which depends on the relative velocity between the source and the observer.

The type of symmetry between  $S$  and  $M$  which was explored in the case at which time was taken absolute, is now to be restricted to infinitesimal periods of measurements in  $M$ . We shall see that for a process that takes a finite time interval the symmetry between  $S$  and  $M$  ceases to exist, and the observer which is indeed a permanent one is privileged in the sense that his time measurements for the durations of signals coincide with the time indicated by a clock, which in our formulation will turn out to be absolute.

## 3. Longitudinal Motion

Consider a source of light  $B$  moving relative to the inertial frame  $S \equiv (S, o)$  along the  $x$ -axis with constant velocity  $\vec{u} = u\vec{i}$ , where  $\vec{i}$  is the unit vector of the  $x$ -axis, and suppose that  $B$  is approaching  $o$  from left. The light emitted by the source  $B$  and received latter by  $o$  at some instant  $t = 0$ , may be envisaged by the observer  $o$ , as well as by any  $S$ -observer, in either of the following ways:

(i) Light has been emitted from the source  $B$  and in accord with the Galilean law of velocity addition, it should acquire a velocity  $c + u$  relative to  $o$ , and to any  $S$ -observer.

(ii) Light has been emitted from the stationary point  $b$  in  $S$ , which was occupied by the source at an earlier instant  $t = x_0 / c$ , where  $x_0$  is the coordinate of  $b$ .

The pulse takes by the second point of view a duration  $\Delta t = -x_0 / c$  to reach  $o$  and  $\Delta T = -x_0 / (c + u)$  by the first view. Thus we have

$$-x_0 = c\Delta t = (c + u)\Delta T$$

and hence

$$(3.1) \quad \Delta t = (1 + u/c)\Delta T$$

This current view is unaltered had we introduced a true source of light fixed at  $b$  that sends a pulse of a very short duration as the source  $B$  passes through the point  $b$ . We shall describe such a source as virtual, although it may be a true source.

As  $o$  is not in a better status than any other inertial observer, the inertial observer  $O$  who is moving relative to  $S$  with velocity  $\vec{u} = u\vec{i}$  and just passing by  $o$  when light reaches  $o$  (and of course  $O$ ) is equally entitled to specify the relation between  $\Delta t$  and  $\Delta T$ . Suppose that the observer  $O$  is endowed with an inertial frame  $M \equiv M_0 \equiv OXY$  in standard configuration with  $S$ . Relative to  $M$  the source  $B$  is stationary, whereas its location  $b$  is moving away from  $O$  with velocity  $(-u)$ . Using the Galilean law of velocity addition, the observer  $O$  deduces that the time intervals  $\Delta t$  and  $\Delta T$  considered above, are related by the equation  $(c - u)\Delta t = c\Delta T$ , or

$$(3.2) \quad \Delta T = (1 - u/c)\Delta t.$$

Being physical quantities for both observers  $O$  and  $o$ , the intervals  $\Delta T$  and  $\Delta t$  refer to the same quantities in equations (3.1) and (3.2). Indeed  $\Delta T$  is the time interval taken by light if emitted from the location  $b$  to reach  $O$  and  $o$ , whereas  $\Delta t$  is such time interval if light is emitted from the source  $B$ . However, and if we substitute for  $\Delta T$  and  $\Delta t$  from one equation into the other, we obtain

$$\Delta T = (1 - \beta^2)\Delta t, \quad \Delta t = (1 - \beta^2)\Delta T$$

where  $\beta = u/c$ . To resolve this contradiction and maintain at the same time the symmetric status of the observer

$O$  and  $o$ , we have to scale the right hand-sides of equations (3.1) and (3.2) through multiplying by  $1/\sqrt{1 - \beta^2}$ . This scaling process yields the relations

$$(3.4) \quad \Delta t = \sqrt{\frac{1 + \beta}{1 - \beta}}\Delta T, \quad \Delta T = \sqrt{\frac{1 - \beta}{1 + \beta}}\Delta t.$$

Setting

$$(3.5) \quad \gamma = \sqrt{\frac{1 + \beta}{1 - \beta}},$$

we write the latter equations as

$$(3.6) \quad \Delta t = \gamma \Delta T, \quad \Delta T = \gamma^{-1} \Delta t.$$

Utilizing the assumption that the velocity of light within each inertial frame is a constant  $c$ , and multiplying the latter equations by  $(-c)$  we obtain

$$(3.7) \quad x_0 = \gamma X_0, \quad X_0 = \gamma^{-1} x_0,$$

where  $(-x_0)$  is the distance traveled by light from the location  $b$  to  $o$  (or  $O$ ) and  $(-X_0)$  is the distance traveled by light from the source  $B$  to  $O$  (or  $o$  which is just passing by  $O$  when light reaches  $O$ ). The relations (3.6) and (3.7) hold as long as  $B$  approaches  $o$ . However, as  $b$  bypasses  $o$  and travels away, these relations become

$$\text{are (3.8) } \Delta t = \gamma^{-1} \Delta T, \quad \Delta T = \gamma \Delta t$$

$$(3.9) \quad x_0 = \gamma^{-1} X_0, \quad X_0 = \gamma x_0.$$

It must be noted that the equations (3.6) guarantee simultaneous arrival of light from  $B$  and  $b$  at  $o$  (and  $O$ ). Indeed and according to the observers  $o$  and  $O$  the emission of light from  $B$  occurs at  $T = X_0/c$ , and it takes light a duration  $\Delta T = -X_0/c$  to reach  $o$  and  $O$  at  $X_0/c + (-X_0/c) = 0$ . Also both observers assign to the event of light emission from  $b$  the instant  $t = x_0/c$ , and the flight time  $\Delta t = -x_0/c$ , which amounts to reaching  $O$  and  $o$  at  $x_0/c + (-x_0/c) = 0$ .

An observer, such as  $o$ , who with respect to his location the measurement of the time interval  $(B \rightarrow o)$  and the evaluation of the corresponding time interval  $(b \rightarrow o)$  are carried out, will be called an "active observer". It is important to see that the S- as well as the M-observers agree all with the active observer  $o$  measurements and evaluations. It is noted that the nature of  $b$  as a true or a hypothetical source has no effect on the results we have obtained. We may thus consider the previous treatment as involving a system of two sources, one of each is stationary in one frame and moving in the other. The true source approaches  $o$  while the virtual source moves away from  $O$ . The observers  $O$  and  $o$  associate with each body (source), or more precisely with the distance separating him from the source, a certain time interval  $\Delta T$  or  $\Delta t$ , so that the same time interval is associated with the same body. Both observers  $O$  and  $o$  agree that: the time intervals  $(B \rightarrow O), (B \rightarrow o)$  equal to  $\Delta T$ ; the time intervals  $(b \rightarrow O), (b \rightarrow o)$  are equal to  $\Delta t$ , and that  $\Delta t = \gamma \Delta T$ , provided that  $B$  is approaching  $o$ . An equivalent alternative view is the following: denote by  $d = -x_0$  the distance between  $b$  and  $o$  (or  $O$ ) and by  $D_0 = -X_0$  the distance from  $B$  to  $O$  (or  $o$ ), then  $d = \gamma D_0$ . The observer  $o$  finds that the moving body  $B$  (towards  $o$ ) is closer than its location  $b$  by a factor  $1/\gamma$ . Similarly,  $O$  finds the distance of the virtual source  $b$ , which is moving away from  $O$ , is larger than the distance of its location  $B$  by a factor  $\gamma$ .

Instead of considering the body  $B$  as moving relative to  $(S, o)$ , we may assume that  $o$  is moving relative to the

frame  $M$  in which  $B$  is momentarily stationary, and we get the same result: the body  $B$  seems closer to  $o$  (and  $O$ ) than its location  $b$ . In this concern, it is helpful and constructive to visualize each of the two systems (body  $B$  and observer  $O$ ) and (location  $b$  and observer  $o$ ) as two rigid rulers sliding on each other in opposite directions, and it is under our disposal to adopt either of the following points views: (i)  $bo$  is sliding on the stationary ruler  $BO$ , and when light initiated from  $B$  (or  $b$ ) the observer  $o$  was to the right of  $O$ , (ii)  $bo$  is stationary whereas  $BO$  is sliding on  $bo$ , and when light was initiated from  $b$  (or  $B$ )  $O$  was to the left of  $o$ .

We shall refer to  $x_0$  and  $X_0$  as the body's stationary and moving coordinates respectively, or in brief, the s- and m-coordinate respectively. We may also distinguish the m-coordinate and time, when the body is approaching or receding from  $o$ , by a prime or double primes on  $m$  respectively.

Note that if we let  $x_0$  and  $X_0$  refer to the s- and m-coordinate of  $B$  at an arbitrary instant  $t$ , then

$$dX_0 / dT = dx_0 / dt = u.$$

Note also that the velocity of light calculated as the quotient of the distance it travels to the relevant time is  $c$ , whether this quotient involves the s-coordinate and s-time, or the m-coordinate and m-time:

$$-X_0 / \Delta T = -x_0 / \Delta t = c$$

#### 4. The Inter-Bodies Time Intervals

Let  $b_1$  be a point of the  $x$ -axis with coordinate  $x_1$ , so that  $x_1 - x_0 = \Lambda > 0$ . Suppose that the source emits its first and second pulse at  $x_0$  and  $x_1$  respectively. Following reasoning similar to that we have just carried out in section 2, we obtain

$$(4.1) \quad x_1 = \gamma X_1 \quad X_1 = \gamma^{-1} x_1,$$

where  $(-x_1)$  is the distance traveled by light had it been emitted from  $b_1$  and till it reaches  $o$  (or  $O_1$  which is a point of  $M \equiv (M, O)$  just passing by  $o$  when light from  $b_1$  reaches  $o$ ) Similarly,  $(-X_1)$  is the distance traveled by light considered as emitted from  $B$  at the position  $b_1$  and till it reaches  $o$  (or  $O_1$ ). Subtracting (3.9) from (4.1) yields

$$(4.2) \quad \Lambda = x_1 - x_0 = \gamma(X_1 - X_0)$$

This shows that the s-distance  $x_1 - x_0$  is  $\gamma$  times greater than the m-distance  $X_1 - X_0$ . In the  $S$ -frame or  $o$ -time, a duration  $t = \Lambda / u$  elapses between the passage of the body  $B$  from  $b$  to  $b_1$ , and hence

$$(4.3) \quad ut = \Lambda = \gamma(X_1 - X_0) = \gamma u T,$$

where  $T$  is the time interval to be associated with the m-distance  $X_1 - X_0$ . From (4.3) we have

$$(4.4) \quad t = \gamma T.$$

The relations (4.4) and (4.2) shows that the m-time and the m-distance of  $B$  from  $o$  are  $1/\gamma$  of the corresponding s-time and s-distance. In other words, and corresponding to a period  $t$  is  $S$ , the duration that  $(S,o)$  associates with the motion of the body during this period is  $T = \gamma^{-1}t$ . It is clear that the relations (4.4) hold for infinitesimals. The latter situations corresponds to  $x_1 - x = dx = udt$ , and hence

$$(4.5) \quad dt = \gamma dT$$

It is tempting to identify the m-time as the body time as endowed by the observer  $o$ . This however, will lead to the following contradiction:

### The Twin Paradox Reversed

Consider both  $B$  and  $o$  as sources and observers, and assume that  $B$  sets out from  $o$  with velocity  $u$  relative to  $o$ , moves to a point at distance  $s$ , and returns with the same velocity to  $o$ . The observer  $o$  register for this trip a period  $2t = 2s/u$ . The m-time of the first stage of the trip ( $B$  is moving away from  $o$ ) is  $T_1 = \gamma t$ , and the m-time of the second stage ( $B$  is going back) is  $T_2 = t/\gamma$ . Thus

$$T_1 + T_2 = \gamma t + t/\gamma = 2t/\sqrt{1-\beta^2}$$

Thus  $o$  finds that  $B$  ages more. Similarly,  $B$  finds the duration of the trip of  $o$  is  $1/\sqrt{1-\beta^2}$  longer than the duration of his own trip, and accordingly finds that  $o$  ages more. This is the twin paradox in special relativity, however with opposite results.

### The Permanent Observer is Privileged

The last paragraph compels us not to put  $t$  and  $T$  on equal footing. We shall accordingly interpret  $T$  as an attribute of the state of the spatial separation between  $B$  and  $o$ , i.e.  $T$  depends on the rate of change in the distance ( $B \rightarrow o$ ). In contrast, the time of the body  $B$  is the same as the time of the observer  $o$  since both can be regarded as inertial labs, or as to say that the time  $t$  is absolute. It is only the durations of signals transmitted from  $B$  in its route to  $o$ . or vice versa, that undergo contraction or expansion by a factor  $\gamma$ . To elaborate a bit more on the latter points we state that a time interval registered by familiar clock is absolute and is consistent with that assigned to the durations of signals transmitted and received within the same inertial frame, i.e. to signals in a static universe. In a static universe (or more accurately in an inertial frame  $(S,o)$ ) a distance  $d$  between two points (stationary particles) can be measured with a ruler as well as by the duration  $t = d/u$  taken by a body moving with a uniform velocity  $u$  to cover the distance between these two points, and the two measurements are consistent ( $d = ut$ ). We shall assume that the clocks in  $S$  are synchronized and that the body  $B$  when at  $b$  sets his clock to be the same as the time indicated by a clock at  $b$ , say  $t_0$ . The body  $B$  can state that  $o$  is approaching him with velocity  $-u$ , and that at  $t = t_0 + d/u$  will meet  $o$ . The reading of the clock at  $o$  when meeting  $B$ , will have the same reading as  $B$ . Both  $B$  and  $o$  associate with there separation a time interval  $\Delta T$  that is shorter than  $\Delta t = d/c$  when approaching each other and greater than  $\Delta t$  when receding from each other. The latter operation results in consistency, since each observer assigns the same m-time interval to a given absolute time interval. .

Reverting to equation (4.3) we see that the observer  $o$  has associated the time interval  $T$  with the space interval  $X_1 - X_0$ . In order that the observer  $O$  should have the same privilege,  $O$  should remain unreplaced while  $o$  slides along in the negative direction, with  $b$  as a true source. In that case, we may present an argument parallel

to that presented at the beginning of this section to deduce  $T$  is absolute, while  $t = \gamma^{-1}T$  is the  $m$ -time associates with the interval  $Ob$ .

We thus conclude that when a body  $B$  is moving relative to an inertial frame  $S$ , the time to be associated with its motion is  $T = \gamma^{-1}t$ , where  $t$  is the absolute time.

If  $x_0$  is the body's stationary coordinate at  $\tau_0 = x_0/c$  and  $x$  is its stationary coordinate at  $t$  ( $t > \tau_0$ ), then

$$(4.6) \quad x = x_0 + u(t - \tau_0)$$

$$(4.7) \quad = \gamma[X_0 + u(T - \tau'_0)],$$

where  $X_0$  is the body's  $m$ -coordinate at  $\tau'_0 = -X_0/c$ . The  $m$ -coordinate  $X$  at an instant  $T$  is

$$(4.8) \quad X = X_0 + u(T - \tau'_0)$$

The relation (3.6) is valid as long as the body is approaching  $o$ . i.e. as long as  $x < 0$ . For  $x > 0$ ,  $\gamma$  has to be replaced in (3.6) by  $1/\gamma$ .

## 5. Velocity Composition

We have mentioned in section 3 that the inertial frame  $(S,o)$  is not in a better status than any other inertial frame that is contiguous to  $S$ , and we have chosen in particular a frame that is  $u$ -contiguous to  $o$ , i.e. the frame in which the body is momentarily at rest at each instant. This choice is quite prudent if we are to explore the role of the full motion of the body on the relation between the absolute time  $t$  and the signals' time  $T$ . In this section we shall consider the effect of partial motion on the latter relations.

Consider now a body  $B$  moving with a constant velocity  $V$  relative to the inertial frame  $M \equiv (M, O)$ , while  $M$  is moving relative to the inertial frame  $(S,o)$  with a uniform velocity  $u$ . Without loss of generality we shall suppose that  $u$  and  $V$  are positive and that the body is approaching  $O$  (or  $o$ ). Let  $x$  and  $X$  be the body's  $s$ -coordinate (the coordinate of its location) in  $S$  and  $M$  respectively. Since the location of the body in  $M$  is drifting in  $S$  with velocity  $u$ , the  $s$ -coordinate  $X$  of the body in  $M$  is also the  $m$ -coordinate of the body in  $S$ , and hence  $x = \gamma(u)X$ . Let  $X'$  be the  $m$ -location of the body in  $M$ . Since  $B$  is moving relative to  $M$  with velocity  $V$ , we have  $X = \gamma(V)X'$ . It follows that

$$\begin{aligned} x &= \gamma(u)\gamma(V)X' \\ &= \frac{\sqrt{1+u/c}}{\sqrt{1-u/c}} \frac{\sqrt{1+V/c}}{\sqrt{1-V/c}} X' = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} X' \end{aligned}$$

where

$$(5.1) \quad v = \frac{u+V}{1+uV/c^2}$$

It is important to note that we have here the observer  $O$  which is  $u$ -contiguous to  $(S,o)$ , and the observer  $O'$

which is  $V$ -contiguous to  $(M, O)$ , and hence the observer  $O'$  is the proper observer who, in conjunction with  $O$ , can determine the impact of the full motion of the body in  $S$  on the relations between  $x$  and  $X'$ . The relation

$$(5.2) \quad x = \sqrt{\frac{1+v/c}{1-v/c}} X'$$

can be temporarily understood in two different ways

(i) In comparison with (3.9),  $v$  which is given by (5.1) must represent the velocity of the body with respect to  $(S, o)$ , and (5.1) is the law of velocity addition in the current theory.

(ii) The velocity of the body relative to  $(S, o)$  is  $u+V$ , which is the velocity to be associated with the signal, is

$$(5.3) \quad C = c(1+uV/c^2)$$

The latter interpretation must be put aside however, since it does not conform to experiment.

## One-Parameter Group of Transformations

We note finally that the transformations from the  $s$ -time to  $m$ -time

$$x = \sqrt{\frac{1+u/c}{1-u/c}} X = f(X, u)$$

form an 1-parameter group in which the identity element corresponds to  $u=0$ , the inverse of an element  $u$  is  $-u$  and the composition of two elements  $u$  and  $V$  is  $u \circ V = v$ , where  $v$  is determined by (5.1).

## The Drag Effect

This effect is explained in the special theory of relativity by making use of the law of velocity addition (5.1), which is in common between the current theory and the special relativity theory (see Rindler, 1977; Mould, 1994).

## 6. The Doppler Effect

Suppose that a source of light is stationary in  $S$  at the point  $x$  and radiating a monochromatic light of frequency  $\nu$  and period  $\tau = 1/\nu$ . This is received at  $o$  as a light of a frequency that is equal also to  $\nu$ : suppose now that the source is in uniform motion with velocity  $u$  relative to  $S$ . The period of light will be  $\tau' = \tau^{-1} \tau$ , and hence

$$\frac{\nu'}{\nu} = \frac{\tau}{\tau'} = \sqrt{\frac{1+u/c}{1-u/c}},$$

which is Doppler's effect.

The last result can also be reached through the following analysis. Had the body been stationary in  $S$ , an  $n$  periods would have covered an interval  $\Delta t = n \tau$  as the body is moving, there corresponds to  $\Delta t'$  the inter-time

interval (or the m-time interval)  $\Delta t' = \gamma^{-1} \Delta t = n(\tau' / \tau)$ , and hence  $\tau' = \tau / \gamma$ . Should the body be moving away from  $o$ , the radiation will have the frequency  $\nu'' = \gamma^{-1} \nu$ .

## 7. Mass, Energy and Momentum

We have seen (eq(3.3)) that the proper time to be associated with the body's uniform motion towards  $o$  is the  $m'$ -time. For a stationary infinitesimal s-time interval  $dt$  there corresponds the infinitesimal  $m'$ -time interval  $dt' = \gamma^{-1} dt$ . We seek now to define tentatively the m-dynamical basic quantities corresponding to its counter basic s-dynamical quantities. The s-quantities will be denoted by lower briefcase letters whereas the m-quantities will be bear the same symbols with briefcase letters, however, with a prime if the body is approaching the active observer, say  $o$ , and with double primes if receding away from  $o$ .

We start with acceleration:

$$(7.1) \quad \vec{a} = \frac{d}{dt} \vec{v} = \frac{d}{dt} \vec{V} = \frac{dT}{dt} \frac{d}{dT} \vec{V} = \gamma^{-1} \vec{A}'$$

Thus

$$(7.2) \quad \vec{a} = \gamma^{-1} \vec{A}'$$

If  $\vec{f}$  is the s-force required to produce the s-acceleration and  $m$  is the s-mass, then by Newton's second law of motion

$$(7.3) \quad \vec{f} = m\vec{a} = m\gamma^{-1}\vec{A}' = M'\vec{A}' = \vec{F}$$

where

$$(7.4) \quad M' = \gamma^{-1}m$$

is the  $m'$ -mass. Assuming  $o$  is an active observer, the acceleration of the particle during an infinitesimal interval  $dt$  during which the particle is receding away is given by

$$(7.5) \quad \vec{A}'' = \frac{d}{dT} \vec{V} = \frac{dt}{dT} \frac{d}{dt} \vec{v} = \gamma \vec{a}$$

The corresponding  $m$ -mass will be

$$(7.6) \quad M'' = \gamma m$$

Assume now that two identical particles, each with an s-mass  $m$  collide elastically at  $o$  with opposite velocities  $\vec{u}$  and  $-\vec{u}$ . In the  $S$ -frame the particles approach each other with opposite vector velocities and reverse their velocities after collision. With respect to the active observer  $o$ , the m-mass of each particle before collision is

$M' = \gamma^{-1}m$ , and after collision is  $M$ . Since the m-mass of each particle is not invariant we search a quantity that pertains to each particle and remains invariant in the collision process. This means that the quantity which we seek is invariant under velocity inversion, or equivalently under the taking the reciprocal of  $\gamma$ . It is clear that a quantity of the sort  $Q = \alpha(\gamma + \gamma^{-1})$ , where  $\alpha$  is a positive real number, satisfies this the property. i.e.  $Q(u) = Q(-u)$ . The quantity

$$(7.7) \quad M = \frac{1}{2}(M' + M'') = \frac{1}{2}m(\gamma + \gamma^{-1}) = \frac{m}{\sqrt{1 - u^2/c^2}}$$

which depends only on the magnitude of the particle's velocity and on its direction tends also to the rest mass  $m$  when the particle's velocity tends to zero, and will be adopted as the m-mass of the particle. In other words, the proper mass that is to be associated with a particle with a rest mass  $m$  which is moving with velocity  $\vec{u}$  is  $M$ . It obvious also that all  $S$ -observer agree with the arguments of the active observer  $o$ .

Having defined the proper mass of a particle by (7.7) the same arguments followed in relativity theory to set up the mass-energy equivalence applies here (Lawden, 1974), leading to relation

$$(7.8) \quad E = Mc^2.$$

The proper or m-momentum of a particle moving with vector velocity  $\vec{u}$  is defined by

$$(7.9) \quad \vec{P} = M\vec{u} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}}$$

where  $m$  is its s-mass. We redefine the force by

$$(7.10) \quad \vec{F} = \frac{d}{dt} \vec{P}$$

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