the general science journal

Fifth Experimental Test of Einstein's General Relativity II

Shi Yong-Cheng

(Shaoxing University, Dept. of mechanical and electronic enginneering
Shaoxing, Zhejiang, 312000, P.R.CHINA .E-mail:shiycgood@126.co

ABSTRACT

The formula which expresses the difference of the trajectory period of a planet with its proper period is deduced. According to this formula , the slowing down effect of a standard clock situated at rest at the surface of the Earth is 4.983 s/10years. If one can make up a clock whose extent of error is smaller than 0.4983 s/every year, then the mythology of the Einstein 's theory of general relativity will be automatically evaporated.

According to Einstein's theory of general relativity, the motion of Earth in the gravitational field of the Sun must leads to the slowing down effect of standard clocks situated at rest at the surface of Earth. Now the calculation of the effect will be an important criteria of the validity of general relativity since there are atomic clocks with high acuracy which may be considered standard clocks.

The equations of the motion of planets in the Sun gravitational field can be described by three equations of first integral as follws [1]

$$\begin{aligned} &e^{\mu}\left(\frac{dt}{d\tau}\right)^2 - e^{-\mu}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2 = 1, \\ &e^{\mu}\frac{dt}{d\tau} = k, \\ &r^2\frac{d\phi}{d\tau} = h, \end{aligned} \tag{1}$$

where

$$e^{\mu} = 1 - \frac{2m}{r},$$

and m to be the gravitation radius of the Sun: m= 1.48 (km)

One had obtained from these equations the following equation of trajectory of planets [1]

$$r = \frac{p}{1 + e\cos\rho\,\phi},$$

where

$$\rho = 1 - \frac{3m^2}{h^2},\ h = \sqrt{mp}.$$

The trajector periods Tc of planets calculated in the Sun frame of reference can be obtained by eliminated the variable r in the equations (1) as follows

$$\mathrm{Tc} = \frac{p^2 k}{h} \int_0^{2\pi/\rho} \frac{d\phi}{(1+ e\cos\rho\phi)^2 \left[1 - 2m(1+ e\cos\rho\phi)/p\right]}. \qquad (2)$$

Introduce a complex variable z = exp (i$\rho\phi$), we obtain

$$\mathrm{Tc} = \frac{p^2 k}{ih\rho} \oint_{|z|=1} \frac{zdz}{(\frac{1}{2}ez^2 + z + \frac{1}{2}e)^2 \left\{1 - \alpha\left[1 + \frac{1}{2}e(z + z^{-1})\right]\right\}}$$

$$= \frac{p^2 k}{ih\alpha\rho}\left(\frac{-2}{e}\right)^3 \oint_{|z|=1} F(z)dz, \qquad (3)$$

where $\alpha = 2m/p << 1$ and

$$F(z) = \frac{z^2}{(z^2 + \frac{2}{e}z + 1)^2 (z^2 + \frac{2}{e^*} + 1)},$$

where

$$e^* = \frac{\alpha}{1-\alpha}e.$$

Put

$$z^2 + \frac{2}{e}z + 1 = 0, \qquad (4)$$

we obtain two roots

$$z_1 = \frac{1}{e}\left(\sqrt{1-e^2} - 1\right), \quad z_2 = -\frac{1}{e}\left(\sqrt{1-e^2} + 1\right). \qquad (5)$$

where $|z_1| < 1$. Put

$$z^2 + \frac{2}{e^*}z + 1 = 0, \qquad (6)$$

we obtain two roots

$$z_3 = \frac{1}{e^*}\left(\sqrt{1+e^{*2}} + 1\right), \; z_4 = \frac{1}{e^*}\left(1 - \sqrt{1-e^{*2}}\right)., \qquad (7)$$

where $|z_4| < 1$.

Now the function F(z) can be written in the form

$$F(z) = \frac{z^2}{(z - z_1)^2 (z - z_2)^2 (z - z_3)(z - z_4)}. \qquad (8)$$

Applied the techniques of complex function integral calculus, we obtain

$$\oint_{|z|=1} F(z)dz = 2\pi i \{res(F(z_1)) + res(F(z_4))\},$$

where

$$res(F(z_1)) = \lim_{z \to z_1} \frac{d}{dz}\left[(z - z_1)^2 F(z)\right] == f(z_1)(2 - g(z_1)),$$

$$f(z_1) = \frac{z_1}{(z_1 - z_2)^2 (z_1 - z_3)(z_1 - z_4)}$$

$$g(z_1) = z_1 \left(\frac{2}{z_1 - z_2} + \frac{1}{z_1 - z_3} + \frac{1}{z_1 - z_4} \right),$$

By means of Eqs.(4) and (6) we obtain

$$(z_1 - z_3)(z_1 - z_4) = z_1^2 - \frac{2}{e^*} z_1 + 1$$

$$= z_1 (z_1 + \frac{1}{z_1} + \frac{2}{e} - (\frac{2}{e} + \frac{2}{e^*})) = -z_1 \left(\frac{2}{e} + \frac{2}{e^*} \right)$$

$$= \frac{2}{e^2 \alpha} \left(1 - \sqrt{1 - e^2} \right)$$

$$\frac{1}{z_1 - z_3} + \frac{1}{z_1 - z_4} = \frac{2z_1 - (z_3 + z_4)}{(z_1 - z_3)(z_1 - z_4)} =$$

$$= \frac{1}{e} \left(\alpha \sqrt{1 - e^2} - 1 \right) \left(\sqrt{1 - e^2} + 1 \right)$$

and then we have

$$f(z_1) = -\frac{e^3 \alpha}{8(1 - e^2)},$$

$$g(z_1) = 2 - \frac{1}{\sqrt{1 - e^2}} - \alpha \sqrt{1 - e^2},$$

$$res(F(z_1)) = -\frac{e^3 \alpha}{8(1 - e^2)^{3/2}} \left[1 + \alpha (1 - e^2) \right]. \qquad (9)$$

Since z_4 to be first order singular point we have

$$res(F(z_4)) = \lim_{z \to z_4} \left[(z - z_4) F(z) \right] = \frac{z_4^2}{(z_4 - z_1)^2 (z_4 - z_2)^2 (z_4 - z_3)}.$$

Considering z_4 to be the root of Eq.(6) we have

$$z_4 + \frac{1}{z_4} - \frac{2}{e^*} = 0,$$

$$(z_4 - z_1)^2 (z_4 - z_2)^2 = z_4^2 \left(z_4 + \frac{1}{z_4} + \frac{2}{e} \right)^2$$

$$= z_4^2 \left(\frac{2}{e^*} + \frac{2}{e} \right)^2 = \frac{4z_4^2}{(\alpha e)^2},$$

$$z_4 - z_3 = -\frac{2}{e^*}\sqrt{1 - e^2},$$

and then

$$res(F(z_4)) = -\frac{1}{8}\frac{\alpha^2 e^* e^2}{\sqrt{1 - e^{*2}}} = -\frac{1}{8}\frac{\alpha^3 e^3}{\sqrt{(1-\alpha)^2 - \alpha^2 e^2}}. \qquad (10)$$

From (3), (9) and (10) we obtain

$$T_c = \frac{2\pi p^2 k}{h\rho\left(1 - e^2\right)^{3/2}}\left[1 + \left(1 - e^2\right)\alpha\right] + o(\alpha), \qquad (11)$$

which can be expressed in the form

$$\text{Tc} = \text{Tn k}\rho^{-1}\left(1 + \alpha(1 - e^2)\right) + \text{o}(\alpha), \qquad (12)$$

where

$$\text{Tn} \overset{def}{=} 2\pi p^2 h^{-1} \Big/ \sqrt{(1 - e^2)^3}. \qquad (13)$$

By considered the second equation in (1), the periods of the proper time of planets can be determined as follows

$$\text{Tp} = \frac{p^2}{h}\int_0^{2\pi/\rho}\frac{d\phi}{(1 + e\cos\rho\phi)^2} = \text{Tn}/\rho, \qquad (14)$$

which is the recorded time by the clocks situated at rest at the surfaces of the planets when which move a cicle around the Sun.

In order to determine the constant k in the formula (12) , we consider the following equation

$$\left(\frac{du}{d\phi}\right)^2 = -\left(1 - k^2\right)\frac{1}{h^2} + \frac{2m}{h^2}u - u^2 + 2mu^2, \qquad (15)$$

which comes from (1)[1] and where $u = 1/r$ .We then have

$$\frac{du}{d\phi} = -\frac{m}{h^2} ep \sin \rho\phi \ .$$

Putting $\phi = 0$, we obtain

$$\frac{du}{d\phi} = 0,\ u = \frac{1+e}{p}, \qquad (16)$$

and then we obtain from (15)

$$k = \sqrt{1 - \frac{1}{2}\alpha\left(1-e^2\right) - \frac{1}{2}\alpha^2(1+e)^3} = 1 - \frac{1}{4}\alpha\left(1-e^2\right) + o(\alpha). \qquad (17)$$

From (12) , (14) and (17) we obtain

$$\Delta T = T_c - T_p = \frac{T_n}{\rho}\left\{\left[1 - \frac{1}{4}\alpha\left(1-e^2\right)\right] \times \left[1 + \alpha\left(1-e^2\right)\right] - 1\right\} + o(\alpha)$$
$$= \frac{3}{4} T_p \frac{2m}{p}\left(1-e^2\right) + o(\alpha) = \frac{3}{2}\left(\frac{m}{a}\right) T_p + o(\alpha)$$

where

$$a = \frac{p}{1-e^2}.$$

We have

$$T_p = T_c - \Delta T,$$
$$\Delta T = \frac{3}{2}\left(\frac{m}{a}\right)\left(T_c - \Delta T\right) + o(\alpha)$$
$$= \frac{3}{2}\left(\frac{m}{a}\right) T_c - o(\alpha)\Delta T + o(\alpha),$$

and then

$$\Delta T \approx \frac{3}{2}\left(\frac{m}{a}\right) T_c . \qquad (18)$$

Take m = 1.48 (km) (the gravitation radius of the Sun), for the Earth ,Tc = $0.316 \cdot 10^8$ s , and a = $1.495 \cdot 10^8$ km[3] , we then obtain

$$\Delta T \approx \frac{3 \times 1.48 km}{2 \times 1.495 \times 10^8 km} \times 3.16 \times 10^7 s = 0.4893 s$$

A atomic clock situated at rest at the surface of the Earth clock will slowing down 4.893 every ten years. If one can make up a clock whose extent of error is smaller than 0.4983 s / every year, then the mythology of the Einstein 's theory of general relativity will be automatically evaporated.

Reference

1. Bergman,P.G., Introduction to the theory of Relativity
2. Fock, V.,The Theory of Space Time and Gravitation (Pergamon Press, Oxford, England,1964)
3. Berkeley Physics Course Vol.1:MECHANICS MeGrav-Hill,1973.