Complementarity

Operators *div* & *curl* applied to the pair of convective algebraic equations separate the vortical and non-vortical field components. The set of four differential equations is thus obtained. The substitution of classical field transformations into this set gives the hybrid transformations. Their analysis again gives Maxwell's set, but operating by transformed EM quantities, with invariant time. This procedure affirms the classical invariance of Maxwell's equations.

1. Introduction

In the first lesson, treating EM interactions, the two static laws have been resolved into the actions and distributions of respective fields, with the two kinetic pairs introduced directly. In the next lessons, the field distributions have been generalized into Maxwell's equations, whereas mutual comparison of the dissimilar fields around the same moving carriers gives the convective algebraic equations. On the other hand, the substitution of the kinetic – by dissimilar static forces – relatively links EM fields. Finally, the combination of the algebraic and differential sets gives the hybrid equations. There arises the question of mutual equivalence in this family of equations.

The algebraic equations operate by EM fields, and differential – by their derivatives. Thus, the derivation of the former, would give the latter set. Though the *curl*-equations have been derived by generalization of the kinetic distributions, they relate the variation of one, by existence of the other field. This reminds of EM interactions caused by motion of the fields and their objects, being expressed through the algebraic equations. The operators *div* & *curl* applied to an algebraic pair give four differential forms, which should be reduced to Maxwell's equations. This demands full cancellation of all the space derivatives of applied speeds. In the convective processes these are the field speeds, and in the relative ones – that of their objects or of a common receptor.

The divergence or longitudinal gradient expresses the dilatation, and curl or transverse gradient – the distortion or rotation of the fields. Thus, the convective reduction demands the *rigid* moving fields, *steady orientated* in space. If, for the macro-fields – due to deformability of the carrier sets – these conditions are uncertain, they may be basefully postulated for ele-

mentary fields. Only the rigid M fields perform distant actions without delay, and their carriers in the rotating bodies represent the gyroscopes of steady orientations. Superposition of the elementary, gives the resulting effects, satisfying all the conditions needed for mentioned reduction. Conditional reduction of the relative forms is realized by conventional restriction of the motion to translation of rigid objects.

2. *Div*-equations

The moving fields produce dissimilar convective inductions, described by the two equations: $\mathbf{H} = \mathbf{V} \times \mathbf{D}$, $\mathbf{E} = \mathbf{B} \times \mathbf{U}$. Application of *div*-operators to these pair gives the following differential forms of non-vortical components of the induced fields:

$$\nabla \cdot \mathbf{H} = \mathbf{D} \cdot \nabla \times \mathbf{V} - \mathbf{V} \cdot \nabla \times \mathbf{D}\,, \tag{1a}$$

$$\nabla \cdot \mathbf{E} = \mathbf{U} \cdot \nabla \times \mathbf{B} - \mathbf{B} \cdot \nabla \times \mathbf{U}\,. \tag{1b}$$

According to the general attitude, the moving carried fields represent the rigid structures steady orientated. Therefore, the space derivatives of their speeds annul, reducing thus the forms:

$$\nabla \cdot \mathbf{H} = -\mathbf{V} \cdot \nabla \times \mathbf{D} = 0\,, \tag{2a}$$

$$\nabla \cdot \mathbf{D} = \varepsilon\mu\, \mathbf{U} \cdot \nabla \times \mathbf{H} = \mathrm{Q}\,. \tag{2b}$$

Scalar product of *curl* and speed of a field expresses the motion along the axis of its vortex, perpendicular to the planes of the field lines. Let us consider apart each of the two EM fields.

Moving electric field – in (2a), as the vortical field in the form of the closed flux tubes, is not fixed to its carriers, and thus its motion can be conditioned by its own internal tensions only. With respect to the circular form of the field lines, these tensions must be exclusively radial, causing respective spreading or shrinking of the contours of the field lines. In absence of axial motion of the electric field, there is not any induction of a non-vortical magnetic field. This fact confirms its exclusively vortical form.

Although moving magnetic field is ever vortical, it is connected to the magnet, as its dipole carrier. At the axial rotation of the magnet, the field is forced to move in direction of its vortex axis, along the vector-potential. In the former generation of EM processes, this field may be determined by the known equation: $\nabla \times \mathbf{H} = \mathbf{J}_{tot}$. This field cause is total magnetization current, irrespective of its structural layer. Equivalent electric charge is thus produced: $\mathrm{Q} = \varepsilon\mu\, \mathbf{U} \cdot \mathbf{J}$. Longitudinal motion of the current, together with the conductor – as the referent frame, produces the charge. This motion is distinct from that of the moving electricity, forming the current itself.

3. *Curl*-equations

Curl-operators applied to the convective EM fields separate their vortical components. The two differential forms thus obtained are to be reduced to respective Maxwell's equations. Let us treat the former, with distinctions of the latter equation:

$$\nabla \times \mathbf{H} = \nabla \times (\mathbf{V} \times \mathbf{D}) =$$

$$= \mathbf{V}\,\nabla \cdot \mathbf{D} + \mathbf{D} \cdot \nabla\,\mathbf{V} - \mathbf{D}\,\nabla \cdot \mathbf{V} - \mathbf{V} \cdot \nabla\,\mathbf{D}\,. \qquad (3)$$

Annulling the moving field derivatives, this form reduces to its first and last terms:

$$\nabla \times \mathbf{H} = \mathbf{V}\,\nabla \cdot \mathbf{D} - \mathbf{V} \cdot \nabla\,\mathbf{D}\,. \qquad (4)$$

The former of the terms obviously expresses the current causing the magnetic field:

$$\mathbf{V}\,\nabla \cdot \mathbf{D} = \mathbf{V}Q = \mathbf{J}\,. \qquad (5)$$

The moving electricity, as the carrier of its field, forms respective current. At a conduction current, as the motion of a free, statically compensated polarity, this polarity carries together its – statically unmanifested – electric field, thus producing the magnetic field. Due to absence of free magnetic poles, respective term from the latter equation annuls.

The latter term in (4) is convective derivative of the moving electric field. At a point at rest, the field variation opposes to the field gradient:

$$\partial_t \mathbf{D} = -\,\mathbf{V} \cdot \nabla\,\mathbf{D}\,. \qquad (6)$$

With respect to the opposite order of the factors – in the latter equation, respective term is opposite. All this gives Maxwell's *curl*-equations:

$$\nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D}\,, \qquad \nabla \times \mathbf{E} = -\,\partial_t \mathbf{B}\,. \qquad (7)$$

Together with the *div*-pair (2) already obtained, full Maxwell's differential set is thus completed. This is the third way of its derivation.

4. Classical invariance

Measuring receptors are the static objects of similar, or kinetic ones – of dissimilar EM fields. At motion of a common receptor, the same object reacts statically on one, and kinetically on the other field. Summary forces represent the *classical field transformations*: $\mathbf{E}' = \mathbf{E} - \mathbf{B} \times \mathbf{v}$, $\mathbf{H}' = \mathbf{H} + \mathbf{D} \times \mathbf{v}$. Each of these equations expresses the sum of the proper static and improper kinetic interactions, in the form of respective equivalent field, as the formal sum of the two nominally same terms. In the case of presence of only one EM field, an equation would be reduced to the free, and the other – to its linear term.

Operators *div* & *curl* applied to these equations, with substitution of the free terms by Maxwell's set, gives the *hybrid* transformations:

$$\nabla \cdot \mathbf{D}' = Q - \varepsilon\mu\, \nabla \cdot (\mathbf{H} \times \mathbf{v}), \tag{8a}$$

$$\nabla \cdot \mathbf{B}' = \varepsilon\mu\, \nabla \cdot (\mathbf{E} \times \mathbf{v}); \tag{8b}$$

$$\nabla \times \mathbf{E}' = -\,\partial_t \mathbf{B} - \nabla \times (\mathbf{B} \times \mathbf{v}), \tag{9a}$$

$$\nabla \times \mathbf{H}' = \partial_t \mathbf{D} + \mathbf{J} + \nabla \times (\mathbf{D} \times \mathbf{v}). \tag{9b}$$

The same result is obtainable by resolution of the transformations per the free terms, with their substitution into respective Maxwell's equations.

With restriction to translation of a rigid receptor, the expanded space derivatives of the vector products reduce to the following terms:

$$\nabla \cdot (\mathbf{F} \times \mathbf{v}) = \mathbf{v} \cdot \nabla \times \mathbf{F}, \tag{10a}$$

$$\nabla \times (\mathbf{F} \times \mathbf{v}) = \mathbf{v} \cdot \nabla\, \mathbf{F} - \mathbf{v}\, \nabla \cdot \mathbf{F}. \tag{10b}$$

Their application to the above equations gives:

$$\nabla \cdot \mathbf{D}' = Q - \varepsilon\mu\, \mathbf{v} \cdot \mathbf{J} = Q', \tag{11a}$$

$$\nabla \cdot \mathbf{B}' = \varepsilon\mu\, \mathbf{v} \cdot \nabla \times \mathbf{E}; \tag{11b}$$

$$\nabla \times \mathbf{E}' = -\,\partial_t \mathbf{B} - \mathbf{v} \cdot \nabla\, \mathbf{B}, \tag{12a}$$

$$\nabla \times \mathbf{H}' = \partial_t \mathbf{D} + \mathbf{v} \cdot \nabla\, \mathbf{D} + \mathbf{J} - \mathbf{v}\, \nabla \cdot \mathbf{D}. \tag{12b}$$

Apart from the explicit transformation of the charge, and of current $\mathbf{J}' = \mathbf{J} - \mathbf{v}Q$, the sum of a local and relative derivatives represents the displacement current transformation: $\partial_t \mathbf{F}' = \partial_t \mathbf{F} + \mathbf{v} \cdot \nabla \mathbf{F}$. The formal magnetic charge in (11b) may be treated as the linear term of the respective charge transformation. Hybrid transformations are thus reduced to Maxwell's set, with transformed variant quantities. These are the two EM fields and their carriers, including both displacement currents. Classical invariance of Maxwell's equations is thus convincingly affirmed.

5. Conclusion

By analysis of the algebraic equations, the differential set is obtained. Though both sets originate from the same distributions, the former is obtained by comparison of the static and kinetic fields, and the latter – by

their separate generalization. Therefore, their final equivalence could not be certainly expected. Its obtaining indicates the same information contained in the field distributions, and in their mutual relations. This fact further points to a unique EM law, expressed by the algebraic or differential set. Particular equations represent respective projections of this law, according to the formal approaches applied.

On the other hand, the application of the classical transformations to Maxwell's set gives the hybrid set of transformations. With variant quantities transformed, their analysis gives the original form. Classical invariance of this set is thus affirmed. Not only that this fact does not deny the relativistic invariance, but represents the basis of it. By complementary deformation of the transverse fields, longitudinal axis and time, the formal relations are fully conserved. The affirmation of the relativistic invariance, in favor of SRT, would not be sufficiently effective without implicit negation of the classical one.

Irrespective of the historical facts, the classical field transformations might be primarily introduced as the necessary condition of reduction of the hybrid into differential equations. The succeeding analysis shows that this condition was not sufficient. Apart from the transformation of the radius vector, and of EM fields – as its direct consequence, electricity and its current – as the field carriers, including both displacement currents, should be transformed. In the similar way, this is the same at the relativistic invariance. In the jargon of modern physics, the conditional invariance is called covariance. Only law of gravitation obeys a non-conditional invariance, but only under the classical condition of invariant mass.