

**PIRT XI**  
**Physical Interpretations of Relativity  
Theory XI**

**Imperial College, London 12-15 September 2008**

**Twin Paradox of Special Relativity**

Sankar Hajra

Calcutta Philosophical Forum

Salt Lake, AC-54, Sector-1, Flat No.A1, Kolkata-700064, India

E-mail: sankarhajra@yahoo.com

Lorentz Transformation Equations (LTE) predict that if an electric dipole stationary in the free space oscillates  $n$  times/sec, then the same dipole must oscillate  $n \times \sqrt{1 - u^2/c^2}$  times/sec when it moves with a velocity  $u$  in that space. Curiously, the same equations also predict that even if the dipole is at rest in the free space and the measuring apparatus moves with the same opposite velocity, then too, the apparatus will record that the dipole is oscillating  $n \times \sqrt{1 - u^2/c^2}$  times/sec. Classical physics does not accept it. In fact, LTE themselves are not at all acceptable from the consideration of classical physics (as real physical equations).

Albert Einstein has accepted LTE and, thereby, has tried to justify the reality of the LTE by his well known principles (assumptions) which constitute the Special Theory of Relativity (STR). Instead of classical time independent of co-ordinates, it uses relative time as a function of co-ordinates and imports equivalent observers to make the frequency-shift phenomenon (derived from LTE in both ways) intelligible from its novel setting.

Abolition of both the preferential observer in free space and classical absolute time makes much difficulty to settle the real time in the clock of each equivalent observer. The constancy of the speed of light to all equivalent observers further complicates the situation.

Though overlooked, relative time, in spite of its supposed good health, struggles hard, just from its birth, to breath at the cruel noose of many an alien unintelligible concepts in spite of palliative measure (of symmetry) adopted by its originator. This is evident in twin paradox. The difficulty arises not from the management of time artificially by its originator; it originates from the absurd assumption of Einstein that all four Lorentz transformation Equations are real.

The paradox centers on the problem of time in case when a man and his twin have a steady relative motion. From the consideration of STR, each twin would claim that the other's clock runs slow compared to the synchronized clock in his own frame.

Such a decision of relativity arouses suspicion. Now, when relativists try hard with sheer pedagogy to justify such a decision and, when the originator of STR makes another theory to resolute the paradox to his own satisfaction, the suspicion grows strong.

Some people [1] suggest that the resolution of twin paradox as presented by the relativists is devoid of any rationality. They are in favor of the rejection of STR

Some relativists [2] has countered the above consideration from the consideration of STR. But, unfortunately, they have evaded the central question relating to the problem.

However, we may clarify here the central question of twin paradox in the following simple examples:

(a) The theory of relativity predicts that the life-span of radioactive particles increases with velocity which has been verified by experiments when the observer with his measuring apparatus is at rest on the surface of the earth and the radioactive particles moves with respect to it. To establish the validity of the theory of relativity, it is the burden of the relativists to show that similar results are confirmed by experiments when the radioactive particles are stationary on earth while the observer with his measuring apparatus steadily moves in the opposite direction on it. In the absence of such a clear-cut experiment, relativists' analysis is meaningless.

(b) Similarly, STR predicts transverse Doppler Effect (time dilation effect) for steadily moving radiating dipoles which has been verified by experiments when the observer with his measuring apparatus is at rest on the surface of the earth and the radiating dipoles move transversely to the observer. To establish the validity of STR, similar results should be confirmed by experiments when the radiating dipoles are at rest on earth

while the observer with his measuring apparatus moves steadily in the opposite direction on this planet.

Relativists should agree with us that any experiment with latest techniques will not detect any of the phenomena (like time-dilation / transverse Doppler's effect) when the radiating dipoles are at rest on earth while the observer with his measuring apparatus moves steadily in the opposite direction, as our classical theory predicts. And they should admit that physics should be based on available experimental data and not on data which could never be verified, nor on data which are expected to be verified later. Unfortunately, relativists are innocently oblivious of these situations. Their 'consistent' phantom world has in no way been proved to be the real world. Therefore, their discussion is a matter of philosophy, but not of physics.

Moreover, it is really shocking that relativists are not at all aware and do not like to be aware that their favorite 'time -dilation' could easily be explained from classical electrodynamics. However, in that case, the confusing time-dilation of the relativists has been replaced by the natural increment of the period of an electromagnetic event due to motion, time remaining the same to all observers as per classical physics. The gist of this explanation is given below:

When a radiating electric dipole moves steadily on earth, the electric and the induced magnetic fields inside the dipole change as per the classical electrodynamics of Heaviside, and, thereby, all electrodynamic phenomena inside the steadily moving dipole, too, change, which at once destroys relativistic time-dilation concept. The classical treatment in brief is as follows:-

Heaviside [3] deduced classically the electric field ( $\mathbf{E}$ ) and the induced magnetic field ( $\mathbf{B}^*$ ) of a steadily moving point charge ( $Q$ ) with a velocity  $u$  in the free space at a point  $P (r, \theta, \phi)$  in spherical polar coordinate where  $r$  is the distance from the origin as under.

$$\mathbf{E} = \frac{Qk^2 \mathbf{r}}{4\pi\epsilon_0 r^3 [1 - (u^2/c^2) \sin^2 \theta]^{3/2}}, \quad (k = \sqrt{1 - u^2/c^2}) \quad (1)$$

$$\mathbf{B}^* = \mathbf{u} \times \mathbf{E} / c^2 \quad (2)$$

where  $\epsilon_0$  and  $\mu_0$  are the permittivity and permeability of free space, and  $c = 1/\sqrt{\mu_0 \epsilon_0}$ .

Relativists should note that the vital Equations (1) and (2) were deduced *first* not by Albert Einstein from relativity as often tacitly claimed, but by Heaviside from the consideration of classical physics in 1888.

The Magnitude of Electromagnetic momentum of a steadily moving point charge could be written from Searle (1897) [4] from the consideration of classical electrodynamics as

$$|P| = 2T / u = Q^2 u / (6\pi\epsilon_0 c^2 k \delta R) \quad [5] \quad (3)$$

(T is the magnetic energy,  $\delta R$  is the radius of the point charge, the direction of momentum being the direction of the velocity of the charge.)

Electromagnetic force acting on a point charge moving steadily in free space at a direction perpendicular to the direction of the uniform electric field operating in free space.

$$F_{\perp} = (|P| / |u|) a_{\perp} = (m_0 / k) a_{\perp} = \gamma m_0 a_{\perp} [Q^2 / (6\pi\epsilon_0 c^2 \delta R) = m_0, \gamma = 1/k] = m a_{\perp} \quad (4)$$

( $a_{\perp}$ , being the acceleration of the point charge in the direction perpendicular to  $u$ ) which implies from the consideration of classical electrodynamics that transverse electromagnetic mass of charges vary with velocity.

However, all those deductions were based on the experiments made on the surface of the earth and extended to free space which may induce some confusion at this stage of discussion. To avoid any confusion, we may say that in true sense, these deductions are applicable on the surface of the moving earth and the high translational motion of this planet with respect to the sun has no effect on those formulations.

Now, using the above four equations, we shall prove the velocity-dependence of frequency and period of oscillations of an electric dipole classically, for which relativists have burdened their theory with complicated ideas unnecessarily.

Let an electric force  $F_0$  (originating from a small charge) drive a point charge back and forth from one end to the other end of a radiating dipole stationary on the surface of the earth. Then from classical electrodynamics,

$$F_0 = -m_0 \omega_0^2 S \quad (5)$$

when the velocity of oscillation is small ( $m_0$  is the electromagnetic mass of the charge in the stationary dipole,  $\omega_0$  is the radian frequency of oscillation of the charge,  $s$  is the separating distance of the dipole).

Now, if the dipole moves with a velocity  $u$  on earth in any direction perpendicular to its direction of oscillations, the electric force and the magnetic force acting on the charge will be respectively from Eqs. (1) and (2), (when  $\theta = 90^\circ$ )  $\gamma F_0$  and  $-(u^2 / c^2) \gamma F_0$ . Therefore, total electromagnetic force acting on the moving charge is

$$F = \gamma F_0 - (u^2 / c^2) \gamma F_0 = F_0 k \quad (6)$$

Now, under the circumstance that the dipole moves on earth and radiates, we have from the consideration of classical electrodynamics,

$$\mathbf{F} = -m\omega^2\mathbf{S} \quad (7)$$

where  $m$  ( $m_0/k = m$ ) is the electromagnetic mass of the charge in the moving dipole,  $\omega$  is the frequency of oscillation of the charge which is moving with a velocity  $u$  on earth with the dipole, and  $\mathbf{F}$  is the electromagnetic force acting on the moving charge.

From Eqs. (4), (5), (6) and (7) for the dipole moving with an uniform velocity on earth in any direction perpendicular to its direction of oscillation we have,

$$\omega = \omega_0 k \quad (8)$$

This explains transverse Doppler's effect from classical physics.

For a dipole stationary on earth,

$$t_0 = 2\pi / \omega_0 \quad (9)$$

where  $t_0$  is the period of oscillation and  $\omega_0$  is the radian frequency. If the same radiating dipole moves with a velocity  $u$  on earth, then for the moving dipole, the period of oscillation  $t$  and radian frequency  $\omega$  satisfy

$$t = 2\pi / \omega \quad (10)$$

Comparing Eqs. (9) and (10) with the Eq. (8) we have,

$$t = \gamma_0 t_0 \quad (11)$$

The equations (8) shows frequency of oscillation of the moving dipole *decreases* and the equation (11) shows that the period of oscillation of a moving electric dipole increases with its velocity on earth.

*This at once destroys 'here is one time', 'there is another time'-concept as well as the twin paradox of relativity.*

However, in this classical approach, there will be no transverse Doppler's effect when the radiating dipole is at rest on earth while the observer with his measuring apparatus moves transversely to the dipole in the opposite direction.

Now, if transverse Doppler's effect is proved in the case cited in the previous paragraph, relativists with their confusing time-dilation concept may insist on continuing such unending metaphysical discussions on the

resolution of the paradox. Otherwise, such an analysis as in [2] seems to be some pedagogical relativistic nonsense.

References and notes: [1] C. S. Unnikrishnan, Current Science **89**, 12, dated 25-12-05. [2] O.G. Grøn, Current Science **92**, 4, 416-418, dated 25-02-07. [3] O. Heaviside, Electrician, Dec.7, 145, (1888). [4] G.F.C Searle, The Phil. Mag., 1897, 340.

[5]  $[\mathbf{P} = \int_{all\ space} (\mathbf{D} \times \mathbf{B}^*) dv \quad \text{and} \quad T = \frac{\epsilon_0 c^2}{2} \int_{all\ space} B^{*2} dv \quad \text{where} \quad \mathbf{P}, \mathbf{D}, \mathbf{B}^*$  are electromagnetic momentum, electric induction vector and induced magnetic field vector respectively,  $T =$  magnetic energy of a steadily moving system of charges and  $dv$  is the infinitesimal volume element in the free space.

Using  $\mathbf{B}^* = (\mathbf{u} \times \mathbf{E})/c^2$  we have,

$$P = 2T/u \quad (i)$$

Searle in 1897 [G.F.C Searle, The Phil. Mag., 1897, 340] has calculated:

$$T = \frac{q^2 u^2}{16\pi\epsilon_0 l c^2} \left\{ \frac{a^2 + l^2}{2l^2} \ln \frac{a+l}{a-l} - \frac{a}{l} \right\} \text{ for a moving charged ellipsoid with the axes}$$

a: b: b when  $a^2 > k^2 b^2$  where  $l^2 = a^2 - k^2 b^2$ . Putting  $a/l = S$ , we have,

$$T = \frac{q^2 u^2 S}{16\pi\epsilon_0 c^2 a} \left\{ (S^2 + 1) \left( \frac{1}{S} + \frac{1}{3S^3} + \dots \right) - S \right\} = \frac{q^2 u^2}{16\pi\epsilon_0 c^2 a} \left( \frac{4}{3} + \frac{1}{3S^2} + \dots \right) = \frac{q^2 u^2}{12\pi\epsilon_0 c^2 kb}$$

When  $S = \infty$ . This corresponds to the Heaviside's Ellipsoid for when  $S = \infty$ ,  $a^2 = k^2 b^2$ . Replacing b by  $\delta R$ , we get

$$T = q^2 u^2 / (12\pi\epsilon_0 c^2 k \delta R) \quad (ii)$$

From the equations (i) & (ii) we have in vector notation,

$$\mathbf{P} = q^2 \mathbf{u} / (6\pi\epsilon_0 c^2 k \delta R) \quad (iii),$$

From which we have,

$$K = mc^2 - m_0 c^2 \quad (iv)]$$