Relativity simplified

by M.H. Butterfield

Whilst aiming to demystify relativity, Prof. Butterfield suggests a basis for explaining many phenomena concerning light and matter

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This simple and unusual approach requires barely A-level mathematics. Starting from the premise that energy has mass, it rederives Einstein's results for clocks and measuring rods in practical terms without making any assumption about light; it explains the existence of light and the various forms of action at a distance via an 'Impact theory'. This approach leads to the concept of a 'spatially distributed single event' which provides a realistic basis for a theory of matter akin to quantum mechanics.

The early Theory of Relativity has changed remarkably little since its early presentation by Einstein[1]. The theory is widely thought difficult to swallow since the basic assumption that the speed of light 'c' is the same irrespective of relative motion of the source and the observer, goes against common sense; "You cannot add to a velocity and get back the original velocity". The fact that the famous law of addition of velocities that is derived from Einstein's Theory, namely

$$w=\frac{u+v}{1+\frac{uv}{c^2}}, \qquad (1)$$

reduces to the usual $w = u+v$ when u and v are small and yields $w = c$ when $u = c$ for any v, is regarded simply as demonstrating that the algebra is consistent but not as answering the problem.

Basic kinematics – mass and energy

We can take the mystery out of relativity if we start again with traditional Newtonian concepts plus a law stating that 'energy has inertial mass'. This is a much more reasonable proposition to accept than the usual one about the absolute nature of the speed of light. We argue that if something increases its energy by virtue of motion, heat content etc., its mass will also increase and it will require a greater force to accelerate it. The idea of conservation of energy and mass is satisfied if the increase of mass is directly proportional to added energy, that is

$$\begin{pmatrix}\text{mass}\\ \text{increase}\end{pmatrix}=(\text{constant}, \lambda)\times\begin{pmatrix}\text{energy}\\ \text{increase}\end{pmatrix}$$

On the basis that nature is essentially simple we use this very simple relationship.

Suppose we take any object, for example a brick, of mass m_0, place it (at rest) on a long smooth straight table and apply a constant force F to accelerate it along the table. When the brick has moved a distance x the force will have done work Fx on it equal to the increase in kinetic energy. This increase of energy Fx appears as an increase in mass λFx; the brick gets more massive and its acceleration reduces. According to Newton, force is rate of change of momentum, so

Consider now the two laboratories L_1 and L_2 moving with relative velocity V and both observing the same brick under prolonged acceleration. Even if the accelerating force is different according to L_1 and L_2 we conclude that both will observe the brick to approach the same asymptotic speed $c = 1/\sqrt{\lambda}$. Hence we have the situation that c + v gives c! Notice that the concept of light has not yet been introduced, nor the principle of relativity, except as it is embodied in Newtonian mechanics.

Light and the 'Impact theory'

The previous section explained that as the clock is accelerated relative to L_1 it is observed to go more slowly according to equation 6. Ultimately on approaching the velocity c its mass would become infinite and it would cease to go round. As seen by L_1, any two events (x_1, t_1),(x_2, t_2) on the path of the moving clock at the origin of L_2 are such that

$$x_1-x_2=c(t_1-t_2).$$

But as seen by L_2

$$x'_1=x'_2=0 \text{ and } t'_1-t'_2=0.$$

Both events are at its origin. Hence in the limit according to L_2 spatial separation is zero and time separation zero: they are the same event!

A photon of light is considered to be a particle of zero rest mass moving at speed c, hence the two points on its track corresponding to its creation and absorption are the same event as observed by the photon. Consider for example the photon created as an electron in a lamp filament is slowed. It is seen at a photocell by knocking out another electron. According to our analysis the emission and absorption of the photon are the same event. The energy exchange can therefore be regarded as a "Direct impact" between the electron in the lamp filament and the electron at the sensor. The whole history of the photon is one event!

Simultaneity and measuring rods

It is essential to understand that the length of any object must be defined in terms of simultaneous measurements at the ends. It is therefore necessary to be clear what is meant by simultaneity or synchronizing clocks. This is another area of misunderstanding in writings on relativity.

The experimenter in L_1 synchronizes identical clocks at A and B by sending a pulse of light from A to B and reflecting it back to A. Then half time between start and finish at A is attributed to the reading of the clock at B when the reflection took place. In this way any number of stationary clocks can be synchronized in the system fully consistently.

The experimenter in L_2 moving at speed v relative to L_1 would go through similar procedures for clocks fixed in the frame. While the experimenter sends a signal forward from C to D which is reflected back to C in L_2, the position of C in L_1 moves. Hence according to L_1 the length of the first leg from C to D is greater than the return distance from reflection at D back to C. Since (as we have established) the speed of light is constant in L_1, L_1 registers the reflection at D after half time. Measuring the length CD, L_1 will see it to be shorter than L_2 since L_1 registers the simultaneous position of D with half time somewhat earlier than L_2. This foreshortening corresponds to the Fitzgerald contraction (ref.2) which was subsequently explained by relativity theory and this analysis also shows the length ratio to be $(1-v^2/c^2)^{1/2}$.

Notice that this result is consistent with the effects of motion on clocks described earlier. Since L_1 sees a measuring rod in L_2 as shortened, its ends will pass a point in L_1 more quickly than if the rod had its full length. Alternatively, as explained earlier for a series of events at a fixed point in L_1, L_1 will record a shorter time interval than L_2, which is a consistent result. This foreshortening is also consistent with the direct impact experienced by a photon, since at the speed c, the photon sees everything else foreshortened to zero length.

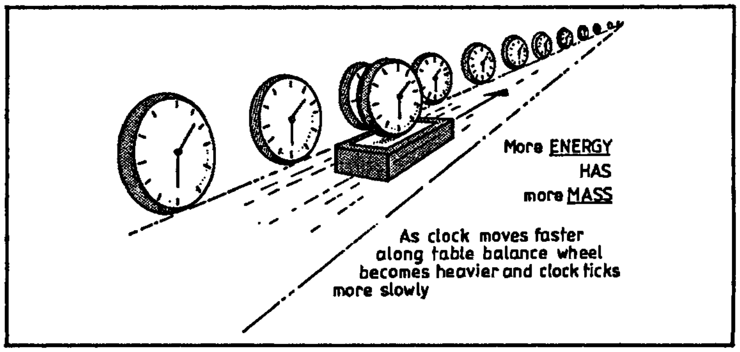
$$F=\frac{d}{dt}\left[m\frac{dx}{dt}\right]=\frac{d}{dt}\left[(m_0+F\lambda x)\frac{dx}{dt}\right] \qquad (2)$$

This equation is easily solved for x as a function of time and gives position,

$$x=\frac{-m_0+\sqrt{m_0{}^2+F^2\lambda t^2}}{F\lambda},$$

and speed,

$$u=\frac{dx}{dt}=\frac{Ft}{\sqrt{m_0{}^2+F^2\lambda t^2}} \qquad (3)$$



As clock moves faster along table balance wheel becomes heavier and clock ticks more slowly

This result has the remarkable property that as t becomes very large the speed tends to $Ft/\sqrt{F^2\lambda t^2}=1/\sqrt{\lambda}$ but this speed is never actually reached. This limiting speed is independent of the initial mass of the brick or the force applied to it, i.e. no material body can be taken beyond (or even quite up to) the speed $1/\sqrt{\lambda}$. If we call this limiting speed c then $\lambda=1/c^2$ and therefore

$$\left(\begin{matrix}\text{energy}\\ \text{increase}\end{matrix}\right)=\left(\begin{matrix}\text{mass}\\ \text{increase}\end{matrix}\right)\times c^2 \quad (4)$$

Now since (3a) and (3b) give

$$1-\frac{u^2}{c^2}=\frac{m_0{}^2}{m_0{}^2+F^2\lambda t^2}=\frac{m_0{}^2}{(m_0+F\lambda x)^2}$$

and $m=m_0+F\lambda x$ as in (2), we get Einstein's formula:

$$m=\frac{m_0}{\left(1-\frac{u^2}{c^2}\right)^{1/2}}. \quad (5)$$

This mass, m, is what an experimenter standing by the side of the long table would measure by displacing the brick sideways, for example.

Time and relative motion

Suppose now the experimenter made a batch of identical clocks with balance wheels of rest mass m_0 and subjected one of them to the accelerating force F. The balance wheel of the moving clock would become more massive as the speed along the table increased and this would alter its observed timing. In practical terms we arrange for the moving clock to spark and burn a hole in the table every time the balance wheel goes through the central position. These 'events' are then timed using the stationary clocks synchronized by conventional means, for example by sending signals or projectiles back and forth.

To quantify the motion of a spring and balance wheel requires relativistic knowledge which we do not have at this stage of the argument. But we can use a simpler mechanism since, by Newton's Laws of Motion, the time spacing must be consistent with a simple disc rotating about an axis parallel to its motion on a frictionless bearing. The mass and hence the moment of inertia of the disc both increase with speed according to equation 5. To maintain constant angular momentum the rate of rotation must therefore reduce as the speed u along the table increases. If the disc made a mark on the bench every time it completed a revolution, the time interval between rotations would increase as the disc moved faster along the table. Hence as seen by the experimenter

$$\left(\begin{matrix}\text{time}\\ \text{interval}\end{matrix}\right)=\frac{\left(\begin{matrix}\text{time}\\ \text{interval}\end{matrix}\right)_{u=0}}{\left(1-\frac{u^2}{c^2}\right)^{1/2}} \quad (6)$$

If the force F is removed on reaching speed u the subsequent motion will be uniform and the speed u of the brick or clock could have been reached via any appropriate history.

Consider now the situation as observed by a second laboratory L_2 moving with constant speed u relative to the first laboratory L_1. The disc was initially travelling backwards at speed u relative to L_2; as seen by L_2 it has been brought to rest by a deceleration force, given up its translational kinetic energy and now has rest mass m_0. Again we can consider the rotating disc marking both tables as it completes each revolution. Initially when the disc is at rest in L_1 it will give spark time intervals T_0 and $T_0(1-u^2/c^2)^{1/2}$ observed by L_1 and L_2 respectively, but when at rest in L_2 the situation is reversed. Hence a clock which is stationary in L_1 appears slow to L_2 and a clock fixed in L_2 appears slow to L_1. These are two different series of events, there is no inconsistency as often presented to provide a *reductio ad absurdum* argument against Einstein's theory of relativity. Both observers see time behaving the same way. It is not the case that clocks are going both faster and slower.

Conclusions to be drawn

1. The existence of light and the absolute nature of its velocity c and the time and space properties of the Lorentz transformation follow directly using Newtonian mechanics from the simple proposition that energy had inertial mass.
2. Along a path covered at the speed c all events are the same and the two ends constitute an 'impact' in that frame of reference. This provides an explanation for electromagnetic radiation and other actions at a distance at the speed of light.
3. Frames of reference with velocity c provide a mechanism whereby every particle in the universe can interact directly with every other particle. While this may seem far-fetched we know that it occurs in universal gravitation. An explanation for gravitational attraction may lie in the notion that the further apart the interacting particles are the more energy is expected to be in transit as we view it; and if the energy of a system increases as particles are separated this shows as an attraction.
4. Since energy is conserved, this continual exchange between all particles may be the mechanism that makes universal constants universal.
5. The idea that the space-time track of a photon is one event yields the concept of a 'spatially distributed single event'. It is not reasonable to consider a fundamental particle such as an electron as a point. The concept evolved here enables an interaction with another particle to occur throughout both as one event. The concept therefore promotes a theory of matter akin to quantum mechanics.
6. The concept of a spatially distributed single event rationalizes interference phenomena. If, as it seems it must, the photon goes through both parallel slits then it is not obviously a single quantum! We can follow the ideas presented by Feynman in ref.3 and consider all the conceptually possible paths collectively as constituting one distributed event. We say that the passages through both slits in the interference experiment are parts of one event and any experiment that we perform to look at this will participate in the event.
7. While the 'Impact theory' describes light as 'corpuscular', it also offers a possible explanation for light to have a wave nature at the same time. According to quantum mechanics particles are subject to a quantization of angular momentum or spin reversal of $h/2\pi$. If we can establish that absorption of light entails spin reversal then the 'Impact' or photon of light as we observe it will have angular momentum $h/2\pi$. If we attribute moment of inertia I and angular velocity $\omega = 2\pi\nu$ to the photon then it has rotational energy:

$$\tfrac{1}{2}I\omega^2=\tfrac{1}{2}(I\omega)\omega=\tfrac{1}{2}.\frac{h}{2\pi}.2\pi\nu$$
$$=\tfrac{1}{2}h\nu.$$

The fore-shortening phenomenon and a single-entity concept for the photon leave open the argument that the photon has only two degrees of freedom (rotation about and translation along its line of motion). If the total energy is equally partitioned we explain the required factor of two to deduce $E = h\nu$.

8. A different type of 'Impact' or photon (for example with different or zero angular momentum) would not be absorbed by intervening matter in the same way as in point 7. This could explain the difference in character between electromagnetic radiation and gravitation which is not influenced by intervening matter.

References

1. The Meaning of Relativity, by A. Einstein. Methuen, 1922.
2. Spare Time and Gravitation, by A.S. Eddington. CUP, 1923.
3. Quantum Mechanics and Path Integrals, by R.P. Feynman and A.R. Hibbs. McGraw-Hill, 1965.

Events

May 23
Use of personal computers in control systems, IEE Colloquium at the Institution of Electrical Engineers. Details from IEE, Savoy Place, London WC2R 0BL. Tel: 01-240 1871 Ext. 269.
May 27
Mass storage devices for computers. IEE Colloquium Details from IEE as above.
UK Skynet IV satellite. IEE Lecture. Details from IEE as above.
May 28
Test equipment for optical communications systems. IEE Colloquium. Details fron IEE as above.
May 29
Opitcal modulators. IEE Colloquium. As above Automated NDT data reduction. IEE Colloquium. As above Hertz and Randall - pioneers of radiation. IEE Lecture.
May 30
Solid State microwave power generation. IEE Colloquium. Details from IE as above.
June 3 to 5
Advanced infrared detectors and systems. Third International Conference at Savoy Place. Details from IEE as above.
June 10 to 12
Networks 86. International computer communications conference and exhibition. Wembley Conference Centr London. Details from Onlin International. Tel: 01-868 4466.
June 24 to 26
Image processing and its applications. IEE and othe International conference at Imperial College, London. Details from IEE, as above.
Broadcast 86. Internation trade fair for film, radio and tv. Messe Frankfurt. Tel: (069) 7575 458.
July 1 to 3
KBS 86; International conference and exhibition o knowledge based systems. Wembley Conference Centr Details from Online as Jun 10. Voice processing. Conference at Wembley Conference Centre. Online, as above.
July 1 to 4
Radio receivers and associated systems. IERE IEE Conference at Univers of N. Wales, Bangor. Detail from IERE. Tel: 01-388 307
July 8 to 10
Cable 86. Brighton Metropole Hotel. Details fr Online.